

Bayesian methods for borrowing information in clinical drug development

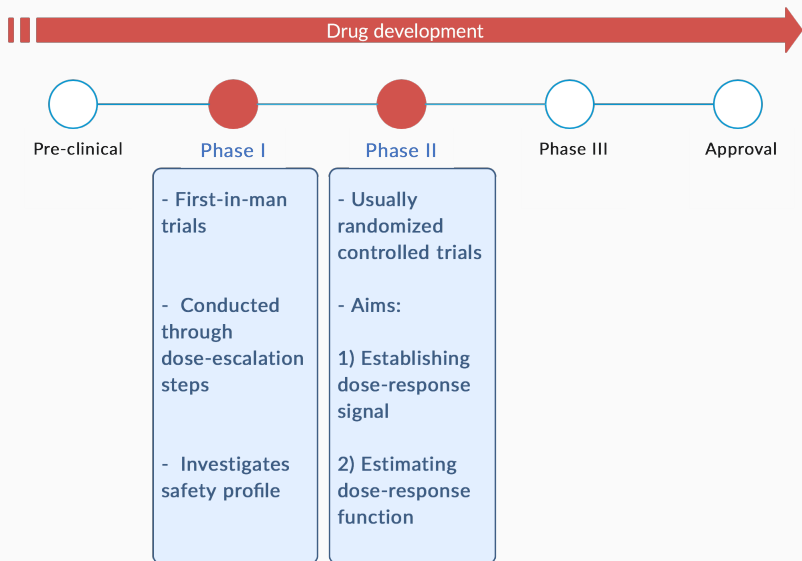
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Introduction

Early phases of clinical drug development

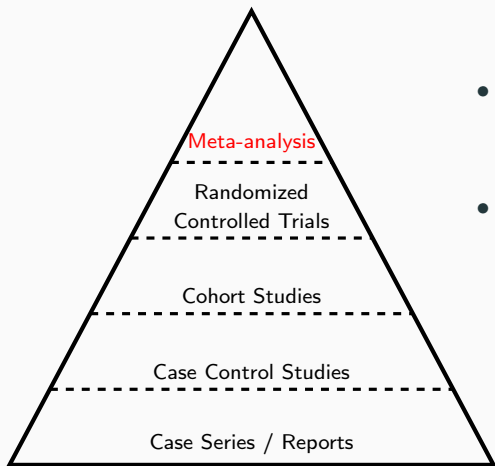


Multiple treatment schedules in phase I and II trials

- **Treatment schedule:** Frequency of administration, e.g. a daily or a weekly schedule
- Recently, phase I or II trials investigating multiple schedules have become more popular, for instance:
 - In oncology (de Lima et al., 2010)
 - In atopic dermatitis (Thaçi et al., 2016)
 - In hypercholesterolemia (Pfizer, 2017)
- Limited literature exists on statistical methods to analyze data from such trials (Guo et al., 2016)

Meta-analysis of clinical trials

- Hierarchy of evidence (Greenhalgh, 1997)



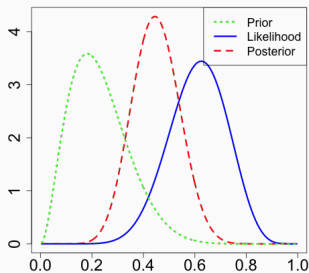
- Statistical methods to combine multiple trials to address a question of interest
- Common methods
 - Fixed-effects meta-analysis
 - Random-effects meta-analysis
- Challenges to conduct a meta-analysis
 - **Rare events**, outcomes with very low event probabilities
 - **Few trials**, e.g. five or fewer trials

Bayesian methods in clinical drug development

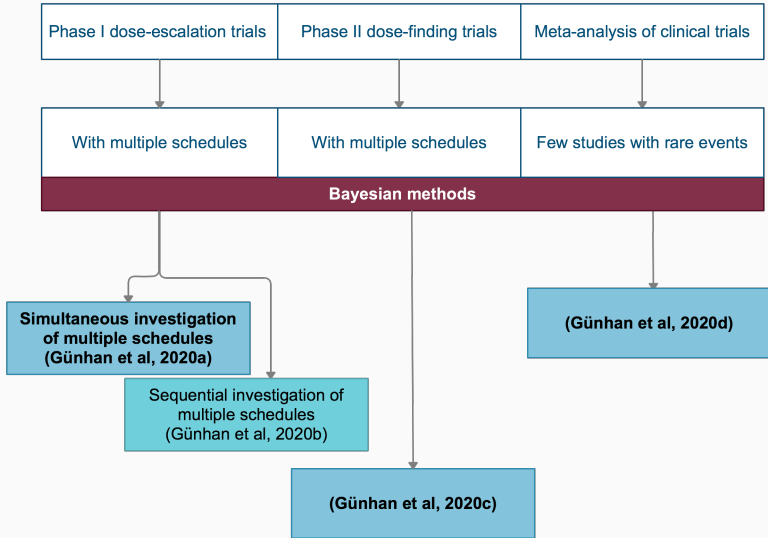
- A particular approach to solve statistical problems
- Bayes' theorem:

$$f(\theta|x) \propto f(x|\theta) f(\theta)$$

- $f(\theta|x)$ the posterior
- $f(x|\theta)$ the likelihood
- $f(\theta)$ the prior
- Bayesian methods used, for example:
 - In phase I dose-escalation trials, Continual Reassessment Method (O'Quigley et al., 1990)
 - In phase II trials (Thomas, 2006)
 - In meta-analysis of clinical trials (Smith et al., 1995)



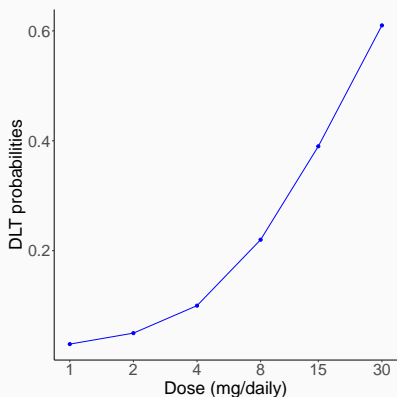
Three major focus areas of the dissertation



**Phase I dose-escalation trials
with multiple schedules
(Günhan et al, 2020a and
Günhan et al, 2020b)**

Phase I dose-escalation trials

- Small cohorts of patients are treated in treatment cycles
- Relationship between dose and probability of dose-limiting-toxicities (DLTs)
- Aim: Informing dose-escalation decisions and finding the **maximum tolerated dose** (MTD)
- With multiple schedules, two types of designs possible



1) Simultaneous investigation of multiple schedules

- Dose and schedule are varied within the same trial
- Finding **maximum tolerated dose and schedule combination (MTC)**

Cohort	Frequency (hrs)	Dose (mg)	Number of pats	Number of DLTs	MTC
1	192	8	1	0	No
2	96	16	1	0	No
3	96	24	1	0	No
⋮	⋮	⋮	⋮	⋮	⋮
30	24	24	1	0	Yes!

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30	24	24	1	0	Yes!

2) Sequential investigation of multiple schedules

Cohort	Frequency (hrs)	Dose (mg)	Number of pats	Number of DLTs	MTD
1	48	2.5	3	0	No
2	48	5	3	0	No
⋮	⋮	⋮	⋮	⋮	⋮
7	48	15	3	1	Yes!

Cohort	Frequency (hrs)	Dose (mg)	Number of pats	Number of DLTs	MTD
8	24	10	3	1	No
9	24	7.5	3	1	No
⋮	⋮	⋮	⋮	⋮	⋮
14	24	7.5	3	0	Yes!

2) Sequential investigation of multiple schedules

Cohort	Frequency (hrs)	Dose (mg)	Number of pats	Number of DLTs	MTD
1	48	2.5	3	0	No
2	48	5	3	0	No
⋮	⋮	⋮	⋮	⋮	⋮
7	48	15	3	1	Yes!

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⋮	⋮	⋮	⋮	⋮	⋮
14	24	7.5	3	0	Yes!

A Bayesian time-to-event pharmacokinetic model (TITE-PK)

- Modelling time to first DLTs, depends on an **exposure** measure $E(t)$
- Exposure measure based on drug **pharmacokinetics** (PK), treated as fixed
- Use of planned schedule and known PK parameter elimination rate constant
- A time-varying Poisson process
- Hazard $h(t)$ and cumulative hazard $H(t)$ given by

$$h(t) = \beta E(t) \implies H(t) = \beta \text{AUC}_E(t)$$

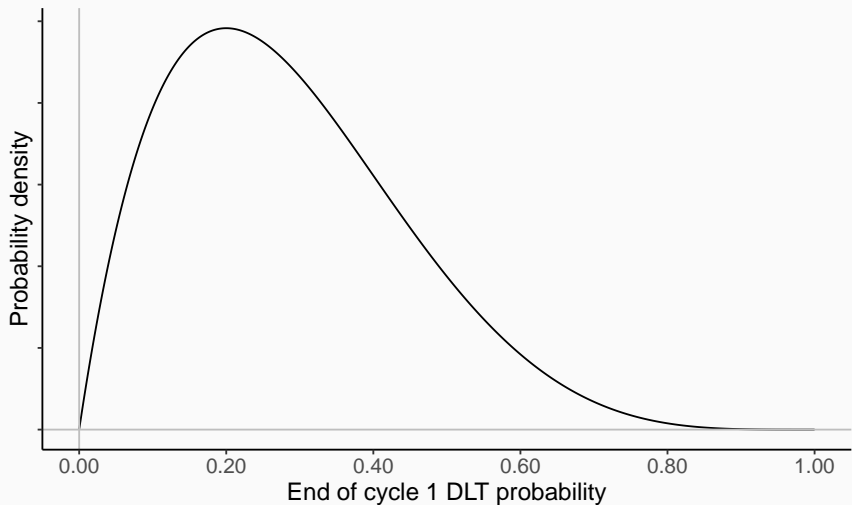
Time-to-event pharmacokinetic model (TITE-PK) (cont.)

- Metric to inform dose-escalation decisions:
End-of-cycle 1 DLT probability

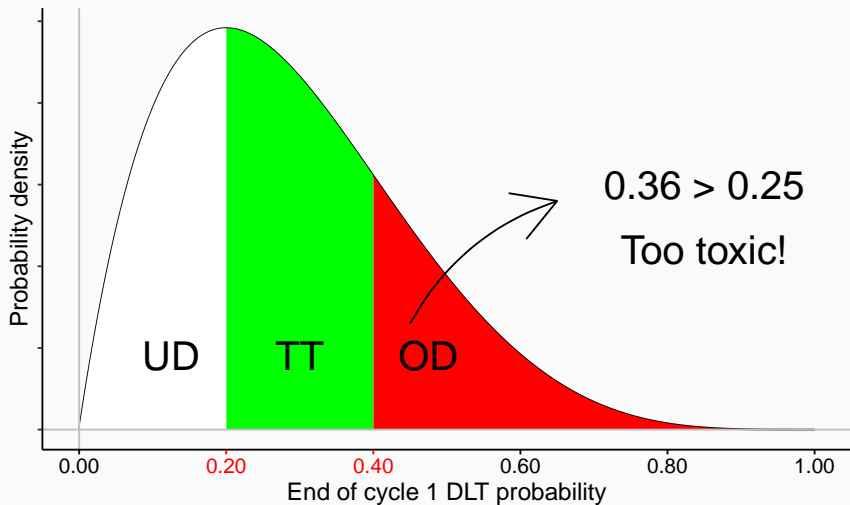
$$P(T \leq t^* | d, f) = 1 - \exp(-H(t^* | d, f))$$

- Three categories for $P(T \leq t^* | d, f)$
 - (i) $P(T \leq t^* | d, f) < 0.20$ Underdosing (UD)
 - (ii) $0.20 \leq P(T \leq t^* | d, f) \leq 0.40$ Targeted toxicity (TT)
 - (iii) $P(T \leq t^* | d, f) > 0.40$ Overdosing (OD)
- **Escalation with overdose control** (EWOC) (Babb et al., 1998)
 - $P(P(T \leq t^* | d, f) \geq 0.40)$ should not exceed 0.25

Visualization of the dose-schedule decisions



Visualization of the dose-schedule decisions



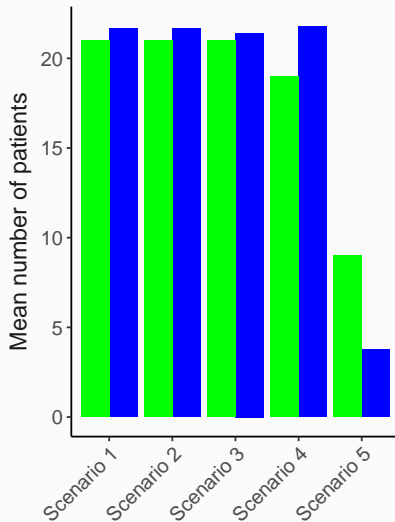
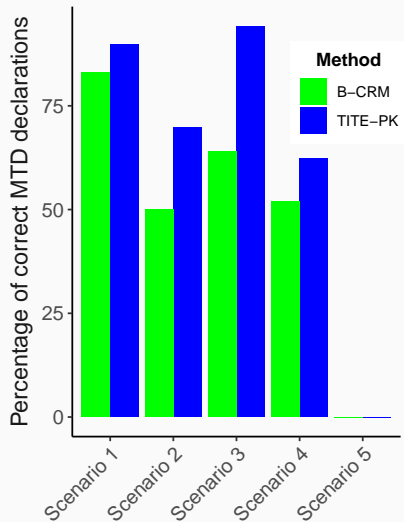
Simulation setup (Sequential design)

- Comparison of performances: TITE-PK vs Bridging Continual Reassessment Method (B-CRM) (Liu et al., 2015)
- B-CRM: Extension of CRM to analyze bridging trials
- Simulations are motivated by the Everolimus trial (NCT00466466)
- Sequential design:
 - S_1 : Once every two days
 - S_2 : Daily
- Performance measures:
 - The percentage of correct MTD declarations
 - Mean number of patients required in the trial

Toxicity probabilities of simulation scenarios

Scenario	Schedule	Doses in mg					
		2.5	5	7.5	10	12.5	15
1	S_1	0.05	0.07	0.09	0.10	0.13	0.18
	S_2	0.08	0.12	0.16	0.18	0.23	0.27
2	S_1	0.08	0.12	0.16	0.20	0.23	0.27
	S_2	0.18	0.26	0.34	0.45	0.49	0.55
3	S_1	0.03	0.12	0.28	0.40	0.54	0.62
	S_2	0.20	0.30	0.45	0.50	0.60	0.75
4	S_1	0.05	0.07	0.09	0.15	0.22	0.28
	S_2	0.30	0.35	0.48	0.52	0.61	0.70
5	S_1	0.45	0.50	0.55	0.65	0.75	0.85
	S_2	0.48	0.56	0.62	0.70	0.80	0.88

Simulation results



Further results for the proposed method TITE-PK

- Sequential investigation of multiple schedules (Günhan et al., 2020b)
 - The Everolimus application is reanalyzed to illustrate TITE-PK
- Simultaneous investigation of multiple schedules (Günhan et al., 2020a)
 - TITE-PK was shown to improve correct MTC declarations with lower sample sizes in simulation studies

Conclusions and outlook

- A time-to-event PK model to analyze phase I trials with multiple schedules
 - Displays better performance in terms of the correct MTD declarations in simulations
- Possible extensions include
 - Modelling multiple compounds (possible interactions must be taken into account)
 - Considering long-term safety events, not only time-to-first DLTs

Phase II dose-finding trials with
multiple schedules (Günhan
et al., 2020c)

- A phase II dose-finding trial: MOR106 investigated for the treatment of atopic dermatitis
- Primary outcome: The percentage change from baseline in Eczema Area and Severity Index at Day 85

Arm	Schedule	Dose (mg/kg)
1	Bi-weekly	0
2	Bi-weekly	1
3	Bi-weekly	3
4	Bi-weekly	10
5	Monthly	1
6	Monthly	3

The general model

- For schedule i , dose j , and patient k

$$y_{ijk} = f(d_j^{(i)}, \theta) + \epsilon_{ijk}, \quad \epsilon_{ijk} \sim \mathcal{N}(0, \sigma_i^2)$$

- The Emax model for dose-response relationship:

$$f(d_j^{(i)}, \theta) = E_0^{(i)} + E_{\max}^{(i)} \frac{d_j^{(i)}}{ED_{50}^{(i)} + d_j^{(i)}}$$

- $E_0^{(i)}$ placebo effect
- $E_{\max}^{(i)}$ maximum effect
- $ED_{50}^{(i)}$ dose providing half of the maximum effect

Different ways to estimate dose-response functions

- Estimating separate curves for each schedule (**stratification**)
 - Ignores the potential similarity in the dose-response functions
- Scaling doses to a common unit and pooling doses from different schedules (**complete pooling**; CP)
 - Ignores the potential heterogeneity in the dose-response functions
- **Partial pooling with fixed-effects** (PP - FE) (Feller et al., 2017)
 - $E_0^{(i)}$ are shared between schedules: $E_0^{(1)} = E_0^{(2)} = \dots$
 - For some situations also $E_{\max}^{(i)}$, but perhaps not $ED_{50}^{(i)}$
 - Using schedule specific fixed-effects for $E_{\max}^{(i)}$ and/or $ED_{50}^{(i)}$

Proposed method: Partial pooling with random-effects

- Schedule specific random-effects for $E_{\max}^{(i)}$ and/or $ED_{50}^{(i)}$ (PP - RE)
- **Exchangeable** around an overall mean
 - $ED_{50}^{(i)}$: Re-scaling and log transformation
 - $ED_{50}^{*(i)} = ED_{50}^{(i)} \frac{f^{(i)}}{f^{(i_{\text{ref}})}}$, $f^{(i)}$ frequency of administration in hours
 - $\log(ED_{50}^{*(i)}) \sim \mathcal{N}(\mu_{ED_{50}}, \tau_{ED_{50}}^2)$

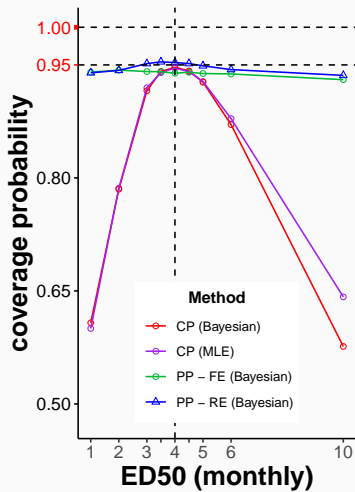
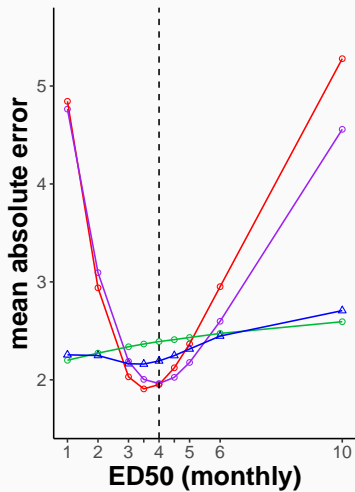
Prior distributions

- Vague priors: $\mathcal{N}(0, 100^2)$ for E_0 and $E_{\max}^{(i)}$ and $\mathcal{HN}(100)$ for σ_i
- Weakly informative prior (WIP) for τ_{ED50} : $\mathcal{HN}(1)$ (Friede et al., 2017)
- In the frequentist framework, usually bounds are imposed on ED_{50} to ensure convergence, e.g. $[0, 1.5 \times \max(d_j^{(i)})]$
- Functional uniform priors for $ED_{50}^{(i)}$ (Bornkamp, 2014):
Uniformly distributed on the potential different shapes of the underlying Emax model

Simulation setup

- The design of each trial and true values for the model parameters are motivated by the MOR106 trial
 - $ED_{50}^{\text{bi-weekly}}$ is 2 mg/kg
 - 9 scenarios: $ED_{50}^{\text{monthly}} \in \{1, 2, 3, 3.5, 4, 4.5, 5, 6, 10 \text{ (mg/kg)}\}$
- Performance measures:
 - Mean absolute error of the point estimates for the dose-response function evaluated at each dose level of a grid
 - Mean coverage probability of the interval estimates evaluated at each dose level of a grid

Simulation results



Conclusions and outlook

- Partial pooling with schedule specific random-effects:
 - yields more robust mean absolute error and higher coverage compared to the alternatives
 - R package which implements the proposed method, ModStan, is publicly available: <https://github.com/gunhanb/ModStan>
- Possible extensions:
 - Taking into account model uncertainty, considering other dose-response models
 - Meta-analysis of dose-response models

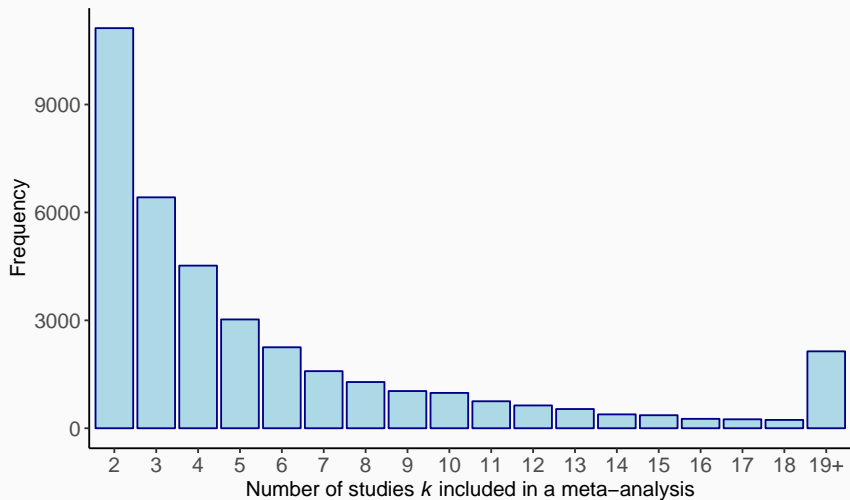
Meta-analysis of few studies
involving rare events (Günhan
et al., 2020d)

- The richest resource of meta-analyses of randomized controlled trials in the world
- I analysed:
 - All datasets available in March 2018
 - Binomial endpoint, both efficacy and safety analyses
 - In total, 37 773 meta-analysis datasets

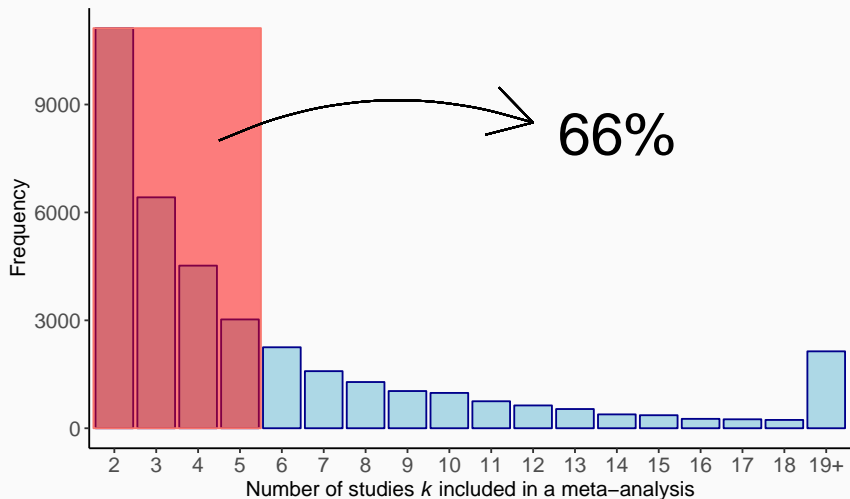
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Cochrane Database: Number of studies



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Arm	Event		Sample size
	Yes	No	
Treatment	a	b	a + b
Control	c	d	c + d

- Percentage of meta-analyses with at least
 - one single-zero study: 38%
 - one double-zero study: 1%
- Standard meta-analysis methods rely on large sample properties (Bradburn et al., 2007)

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The statistical model (Smith et al., 1995)

- For trial i , number of events $r_i \sim \text{Bin}(\pi_i, n_i)$

$$\text{logit}(\pi_i) = \begin{cases} \mu_i - 0.5 \cdot \theta_i & \text{(control arm)} \\ \mu_i + 0.5 \cdot \theta_i & \text{(treatment arm)} \end{cases}$$

- Random treatment effects: $\theta_i \sim \mathcal{N}(\theta, \tau^2)$
- θ on the log-odds ratio scale
- Baseline risks (μ_i) trial-specific fixed-effects

A weakly informative prior (WIP) for θ

- A WIP for heterogeneity parameter τ , e.g. a half-normal prior
- A priori the odds ratio is with 95% probability confined to a certain range:

$$P(1/\delta < \exp(\theta) < \delta) = 95\%$$

- Normal prior: $\sigma = \frac{\log(\delta)}{1.96}$
- Say, conservatively, $\delta = 250$ implies $\theta \sim \mathcal{N}(0, 2.82^2)$
- The use of a WIP may be interpreted as a **penalizing** approach (Greenland and Mansournia, 2015)

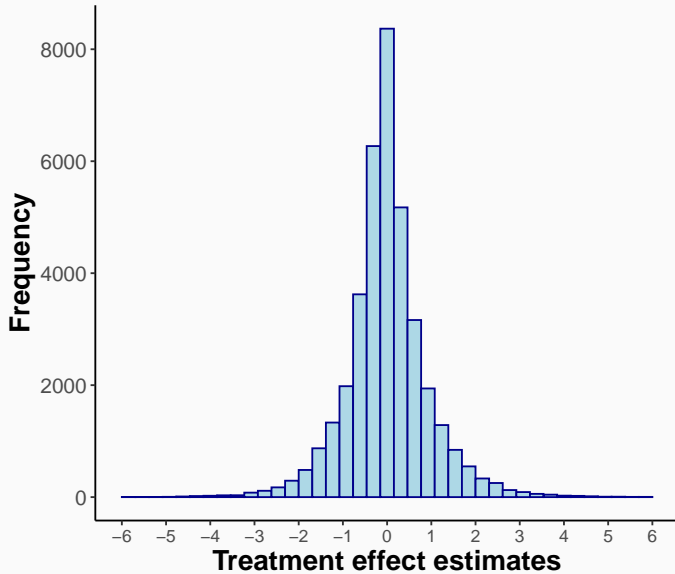
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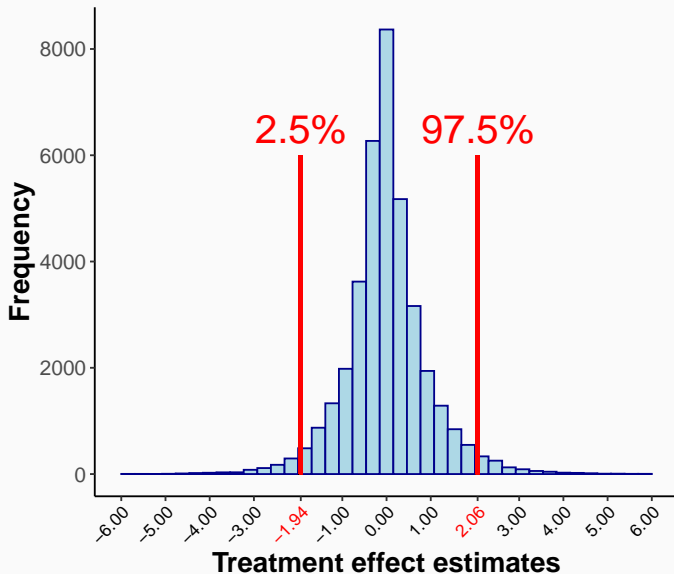
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Cochrane Database: The distribution of the estimates of θ



Cochrane Database: The distribution of the estimates of θ



Further results

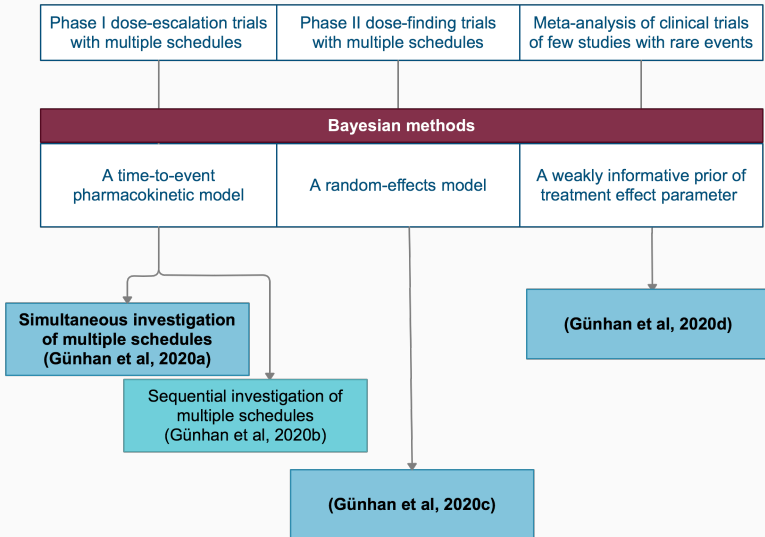
- Simulations are conducted to assess the performance of the use of WIP for θ
- Compared to alternatives, the proposed method displayed
 - lower bias for θ
 - shorter interval estimates for θ with somewhat higher coverage than nominal level
- An R package `MetaStan` is available on CRAN

Discussion and outlook

- Use of WIP for θ and τ for the meta-analysis of few studies involving rare events
 - A WIP can be derived for θ by considering a priori interval for θ on the log-odds ratio scale
 - Empirical investigation from the Cochrane Library supports the proposed WIP
- Future extensions include other type of models, e.g.
 - Poisson-Normal Hierarchical model (Böhning et al., 2015), which can take into account length of follow-up
 - Network meta-analysis (Günhan et al., 2018) models, which investigates multiple treatments and multi-arm trials

Conclusion

Conclusion



Acknowledgements

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- Dr. Christian Röver (UMG, Göttingen)

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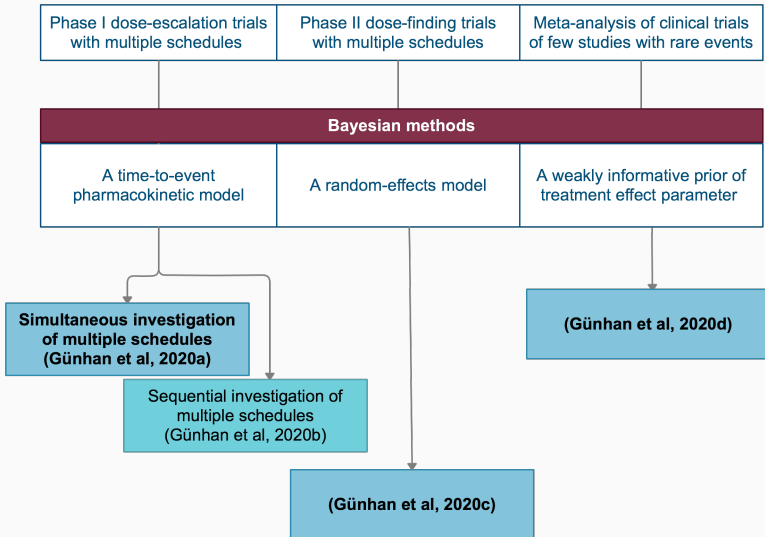
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Conclusion



Stan: A programming language

- Calculating posterior distributions can be hard
Conjugate models (analytically solvable) restrict the choice of likelihood and priors
- Markov chain Monte Carlo methods
Generate samples from posterior distribution
- BUGS-based programs (WinBUGS, JAGS) uses Gibbs and Metropolis-Hasting samplers
- Stan uses Hamiltonian Monte Carlo sampler, uses the geometric nature of the target distribution

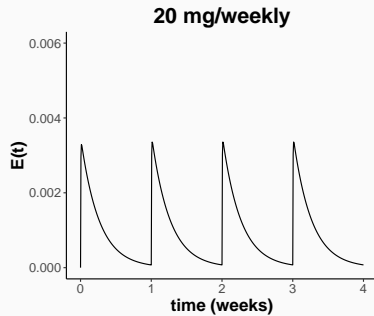
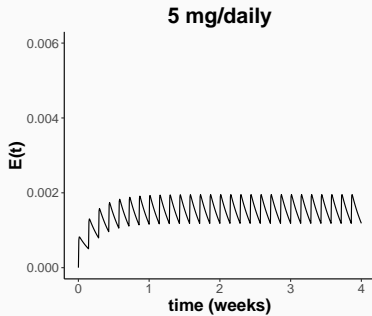
Vaccine trials

- Unlike drugs, which are given to patients, vaccines are received by whole population, thus the safety margin should be very high
- Efficacy based on the protection offered: Binomial endpoint
- Immunogenic endpoint: Antibody concentration, continuous endpoint
- Phase I trials:
 - Oncology vs non-oncology
 - Oncology: The higher the dose, the greater the likelihood of efficacy and toxicity
 - DLT cause the halting of the trial
- Phase II trials:
 - Binomial endpoint: Use of a binomial likelihood
- Rare events:
 - Binomial endpoint
 - Based on the prevalence, we may encounter rare events

Likelihood estimation

- Firth penalization:
 - Penalty terms may be specified so that these nudge the MLE into a desired direction if the maximum is not or poorly defined; one such example is Firth penalization (Firth, 1993)
- Random-effects meta-analysis
 - Bayesian modal estimation to avoid zero estimates (Chung et al, 2013)

Pseudo-PK model



Pseudo-PK model

What body does to the drug

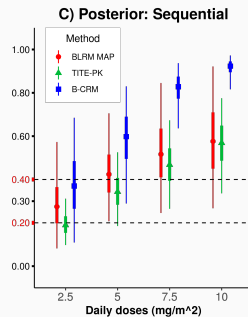
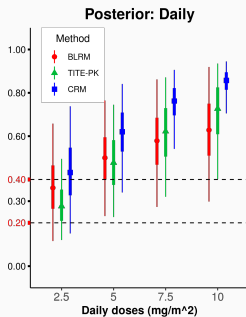
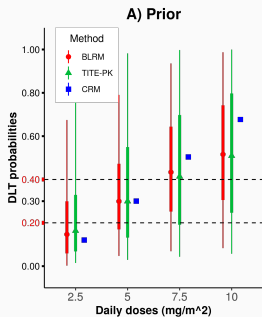
$$\frac{dC(t)}{dt} = -k_e C(t),$$

$$E(t|d, f) = \frac{C_{\text{eff}}(t|d, f)}{\int_0^{t^*} C_{\text{eff}}(t|d^*, f^*) dt}$$

$$\text{AUC}_E(t^*|d^*, f^*) = \int_0^{t^*} E(t|d^*, f^*) dt = 1.$$

- TITE-PK: $\log(\beta) \sim \mathcal{N}(\text{cloglog}(P(T \leq t^* | d^*, f^*) = 0.30), 1.25^2)$
where $\text{cloglog}(x) = \log(-\log(1-x))$

Everolimus trial



Dose-escalation decision criteria

- Cohort sizes of 3
- Next dose / Current dose ≤ 2
- Minimum number of patients at MTD: 6
- Maximum number of patients: 60
- Minimum number of patients: 21
- MTD declaration: $P(OD) \leq 0.25$

WIP for the heterogeneity parameter

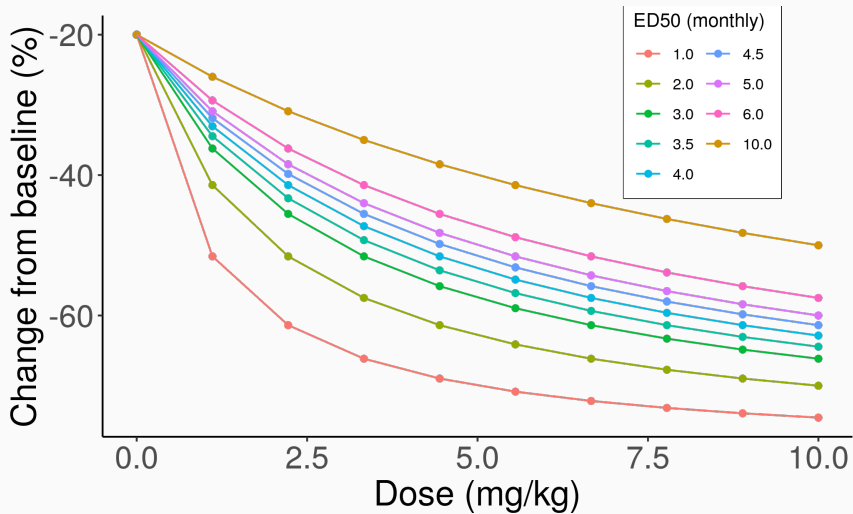
Table 2: Between-schedule heterogeneity $\tau_{ED_{50}}$ in $\log(ED_{50}^{*(i)})$: $\tau_{ED_{50}}$ referring small to very large heterogeneity. The "range", $\exp(3.92 \cdot \tau_{ED_{50}})$, refers to the ratio of the 97.5% to the 2.5% point of the distribution of $ED_{50}^{*(i)}$.

$\tau_{ED_{50}}$	"range" of $ED_{50}^{*(i)}$
0.125 (small)	1.63
0.25 (moderate)	2.66
0.5 (substantial)	7.10
1 (large)	50.40
2 (very large)	2540.20

Simulation setup

- Motivated by the MOR106 trial
- Each generated trial includes one placebo arm and 1, 3, and 10 mg/kg for both bi-weekly and monthly schedules.
- Outcome: Percentage change from baseline in EASI score
- True values for $E_0^{(i)}$, $E_{\max}^{(i)}$ and σ_i are taken as -20%, -60%, and 35% for both schedules, respectively.
- $ED_{50}^{\text{bi-weekly}}$ is 2 mg/kg.
- Sample sizes 45 for each arm.
- 9 scenarios: $ED_{50}^{\text{monthly}} \in \{1, 2, 3, 3.5, 4, 4.5, 5, 6, 10 \text{ (mg/kg)}\}$
- Data-generating process: Emax model
- 1 000 replications

Scenarios (monthly schedule)



Comparison of four methods

1. **CP (Frequentist)**: Complete pooling using a frequentist framework
2. **CP (Bayesian)**: Complete pooling using a Bayesian framework
3. **PP - FE**: Partial pooling with schedule specific fixed-effects for $ED_{50}^{(i)}$ using a Bayesian framework
4. **PP - RE**: Partial pooling with schedule specific random-effects for $ED_{50}^{(i)}$ using a Bayesian framework

Performance measures for the biweekly schedule

1. Mean absolute error

$1/|D| \sum_{i \in D} |f(i) - \hat{f}(i)|$ at each i (prespecified dose levels), $\hat{f}(i)$ the point estimates for the dose-response function $f(i)$

2. Mean coverage probability

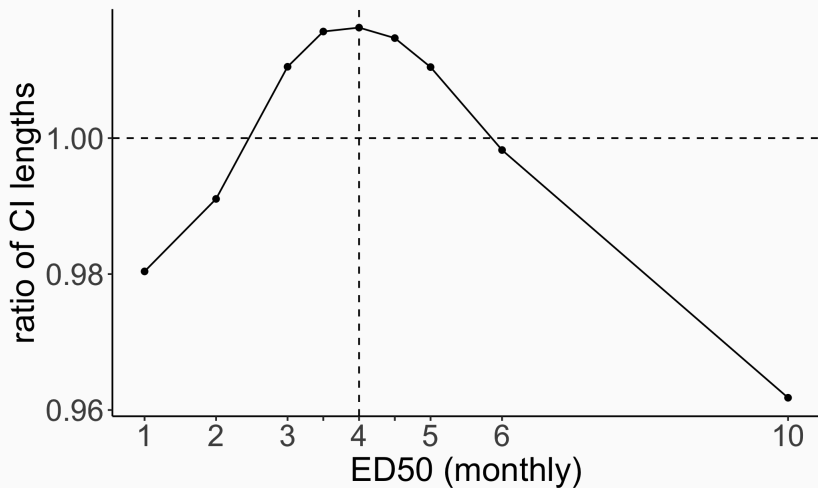
Mean coverage probability of the interval estimates evaluated at each i

3. Mean interval length

Mean length of the interval estimates at each i

Simulation results

- Ratio of the CI lengths obtained by PP - FE to PP - RE



Bococizumab trial: NCT01592240

- A phase II dose-finding trial: Bococizumab investigated for the treatment of hypercholesterolemia
- Primary outcome: The change from baseline in low-density lipoprotein cholesterol (LDL-C) at Day 85

Arm	Schedule	Dose (mg)
1	Bi-weekly	0
2	Bi-weekly	50
3	Bi-weekly	100
4	Bi-weekly	150
5	Monthly	0
6	Monthly	200
7	Monthly	300

Simulation settings

- Numbers of studies:

$$k \in \{2, 3, 5\}$$

- Treatment effects:

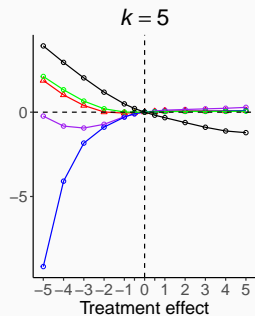
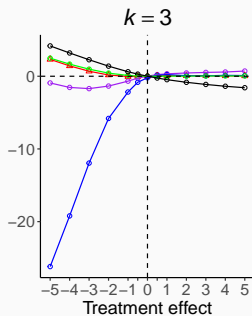
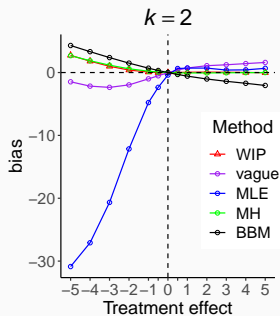
$$\theta \in \{-5, -4, -3, -2, -1, -0.5, 0, 0.5, 1, 2, 3, 4, 5\}$$

- Baseline risks μ_i on the probability scale are taken uniformly between 0.005 and 0.05.
- The degree of heterogeneity ($\tau = 0.28$) and sample sizes are based on Cochrane Database.
- Data-generating process: BNHM

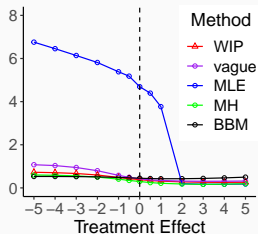
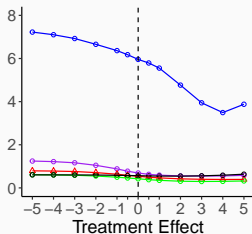
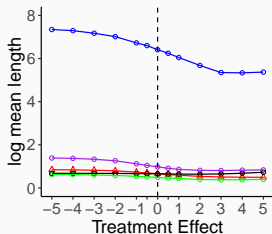
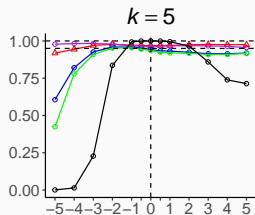
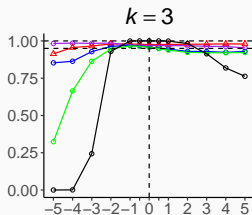
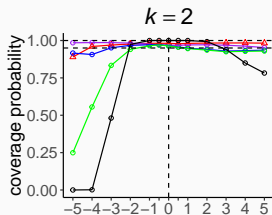
Comparison of five meta-analysis methods

1. WIP: WIP for τ and WIP for θ , BNHM
2. vague: WIP for τ and vague prior for θ ($\theta \sim \mathcal{N}(0, 100^2)$), BNHM
3. MLE: BNHM
4. MH: Mantel-Haenszel method, a fixed-effect method
5. BBM: Beta-binomial model (Bayesian)

Simulation results



Simulation results



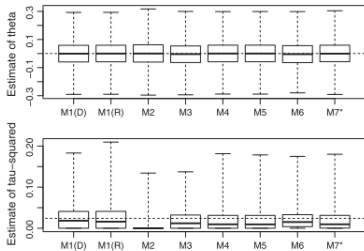
Effects of treatment coding (Jackson et al., 2018)

we have

$$\begin{pmatrix} \text{logit}(\pi_{i0}) \\ \text{logit}(\pi_{i1}) \end{pmatrix} \sim N \left(\begin{pmatrix} \gamma_i \\ \gamma_i + \theta \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & \tau^2 \end{pmatrix} \right),$$

and from Equation 2, for model 4, we have

$$\begin{pmatrix} \text{logit}(\pi_{i0}) \\ \text{logit}(\pi_{i1}) \end{pmatrix} \sim N \left(\begin{pmatrix} \gamma_i \\ \gamma_i + \theta \end{pmatrix}, \begin{pmatrix} \tau^2/4 & -\tau^2/4 \\ -\tau^2/4 & \tau^2/4 \end{pmatrix} \right).$$



An R package for meta-analysis using Stan: MetaStan

Available on CRAN

```
install.packages("MetaStan")
```

Fitting a BNHM using WIP for θ and τ

```
meta_stan(data = mydata,  
          nctrl = nctrl,  
          rctrl = rctrl,  
          ntrt = ntrt,  
          rtrt = rtrt,  
          tau_prior_dist = "half-normal",  
          tau_prior = 0.5,  
          delta = 250)
```

```
vignette("MetaStan_BNHM")
```